**Introduction**

After 10 years of schooling at primary and secondary level, students (16+) who succeed in passing the Secondary School Certificate (SSC) examination have the option of joining a college for a two-year higher secondary education in their respective areas of specialization or enrolling in a technical or polytechnic institute. After the two-year higher secondary education, one has to sit for another public examination called Higher Secondary Certificate Examination conducted by the education boards to qualify for further education. Students of religious and English medium streams also sit for their respective public examinations, Alim and "A" level, conducted by the Madrasah Education Board and London/Cambridge University respectively to qualify for further education. Every year, HSC Exam starts from April in Bangladesh. HSC Exam Routine publishes 2 month ago before examination.

To interpolate the passing rate of SSC examination for the year 2010 we use Lagranrange’s interpolation method.

**Lagranrange’s interpolation method**

Unequally spaced interpolation requires the use of the divided difference formula. It is defined as

*f*(*x, x*0) = *f*(*x*) *− f*(*x*0)

*x − x*0……….(1)

*f*(*x, x*0*, x*1) = *f*(*x, x*0) *− f*(*x*0*, x*1)

*x − x*1…………….(2)

*f*(*x, x*0*, x*1*, x*2) = *f*(*x, x*0*, x*1) *− f*(*x*0*, x*1*, x*2)

*x − x*2

(3)

From equation (2), the formula can be rewritten as

(*x − x*1) *f*(*x, x*0*, x*1) + *f*(*x*0*, x*1) = *f*(*x, x*0) *,*

and the substitution of equation (1) yields,

(*x − x*0)(*x − x*1) *f*(*x, x*0*, x*1) + (*x − x*0) *f*(*x*0*, x*1) + *f*(*x*0) = *f*(*x*) *.*

The first term is considered the remainder term as it is not in the difference table, so *f*(*x*) can be

expressed approximately in terms of the divided differences as

*f*(*x*) *≈ f*(*x*0) + (*x − x*0) *f*(*x*0*, x*1) + (*x − x*0)(*x − x*1) *f*(*x*0*, x*1*, x*2) *,*

a second order formula. The first order formula can be written as

*f*(*x*) *≈ f*(*x*0) + (*x − x*0) *f*(*x*0*, x*1) *.*

The above formulas are the most convenient for numerical computation when the divided differences

are store in a matrix form. But actual explicit formulas can be written in terms of the sample function

Values.

Lagrange First Order Interpolation Formula:

Given

*f*(*x*) = *f*(*x*0) + (*x − x*0)*f*(*x*0) *− f*(*x*1)

*x*0 *− x*1

*.*

Use simplified notations *f*0 = *f*(*x*0), *f*1 = *f*(*x*1), to write

*f*(*x*) = *f*0 +(*x − x*0)(*x*1 *− x*0)(*f*1 *− f*0)= *f*0

\_(*x*1 *− x*0) *−* (*x − x*0)(*x*1 *− x*0)\_

+

(*x − x*0)

(*x*1 *− x*0)*f*1

*f*(*x*) =

(*x − x*1)

(*x*0 *− x*1)*f*0 +

(*x − x*0)

(*x*1 *− x*0)*f*1

Lagrange Second Order Interpolation Formula:

Given

*f*(*x*) = *f*(*x*0) + (*x − x*0)*f*(*x*0) *− f*(*x*1)

*x*0 *− x*1

+ (*x − x*0)(*x − x*1)*f*(*x*0*, x*1) *− f*(*x*1*, x*2)

*x*0 *− x*2

*.*

or

*f*(*x*) = *f*0 + (*x − x*0) *f*0 *− f*1

*x*0 *− x*1

+

(*x − x*0)(*x − x*1)

*x*0 *− x*2

\_

*f*0 *− f*1

*x*0 *− x*1

*− f*1 *− f*2

*x*1 *− x*2

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Collecting terms for *f*0, *f*1 and *f*2, and after some tedious algebraic manipulation, the second order

Formula can be written as

*f*(*x*) =

(*x − x*1)(*x − x*2)

(*x*0 *− x*1)(*x*0 *− x*2) *f*0 +

(*x − x*0)(*x − x*2)

(*x*1 *− x*0)(*x*1 *− x*2) *f*1 +

(*x − x*0)(*x − x*1)

(*x*2 *− x*0)(*x*2 *− x*1) *f*2 *.*

Lagrange N-th Order Interpolation Formula:

The N-th order formula can be written in the form:

*f*(*x*) = *f*0*δ*0(*x*) + *f*1*δ*1(*x*) + *. . .* + *fNδN*(*x*) *,*

in which, *δj*(*x*) can be written as

*δj*(*x*) =

\_

*N*

*i*=0;*i\_*=*j*(*x − xi*)

\_

*N*

*i*=0;*i\_*=*j*(*xj − xi*)

Each term of *δj*(*x*) has the required properties such that (a) *δj*(*xi*) = 0 when *i \_*= *j* and (b)

*δj*(*xj*) = 1. The above property ensures *f*(*xj*) = *fj* and none of the other sample values (*fi*,

*i \_*= *j*) participate.

**Math Problem with Solution:**

***Problem:*** *The following table gives the HSC pass rate in Bangladesh five different years. Find the pass rate of 2010 year using Langrange interpolation formula.*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | 2008 | 2011 | 2013 | 2016 | 2018 |
| Pass rate | 76.19 | 75.08 | 74.30 | 74.70 | 66.64 |

***Solution:***

We have,

*x0=2008*, *x1=2011, x2=2013*, *x3=2016,x4=2018 and*

*y0=76.19, y1=75.08, y2=74.30, y3 =74.70, y4=66.64, x=2010*





Now substituting, we get

**

*=78.9853*

**Matlab Code:**

clc

clear

a=[2008 2011 2013 2016 2018];

b=[76.19 75.08 74.30 74.70 66.64];

x0= a(1);

x1=a(2);

x2=a(3);

x3=a(4);

x4=a(5);

y0=b(1);

y1=b(2);

y2=b(3);

y3=b(4);

y4=b(5);

z=input('Enter Value X=');

ans1 = (y0\*(z-x1)\*(z-x2)\*(z-x3)\*(z-x4))/((x0-x1)\*(x0-x2)\*(x0-x3)\*(x0-x4));

ans2 = (y1\*(z-x0)\*(z-x2)\*(z-x3)\*(z-x4))/((x1-x0)\*(x1-x2)\*(x1-x3)\*(x1-x4));

ans3 = (y2\*(z-x0)\*(z-x1)\*(z-x3)\*(z-x4))/((x2-x0)\*(x2-x1)\*(x2-x3)\*(x1-x4));

ans4 = (y3\*(z-x0)\*(z-x1)\*(z-x2)\*(z-x4))/((x3-x0)\*(x3-x1)\*(x3-x2)\*(x1-x4));

ans5 = (y4\*(z-x0)\*(z-x1)\*(z-x2)\*(z-x3))/((x4-x0)\*(x4-x1)\*(x4-x2)\*(x4-x3));

result = ans1+ans2+ans3+ans4+ans5;

disp('Reuslt: ');

disp(result);

**Advantage of Lagrange’s interpolation**

* The higher order problem are more accurate and the interpolated or approximate result converges to the exact solution pretty quickly. It's said that the error decreases 2^(n+1) if you decrease the distance between interpolation points by 2, n here is the order of Lagrange’s polynomial used.
* The formula is very simple and easy to remember.
* There is no need to construct the divided difference table.
* The application of the formula is not speedy.

**Disadvantage of Lagrange’s interpolation:**

* A little more work as on increasing the polynomial order you basically introduce more node points and you have to calculate your approximate solutions at all those points.
* There is always a chance to committing some error.
* The calculation provides no check whether the functional value used the taken correctly or not.

**Conclusion**

Lagrange has a better performance at the boundaries. Which makes it more convenient for real time applications.

**REFFERENCES:**

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